

Transformation #3: Rotations

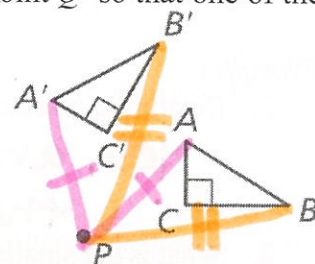
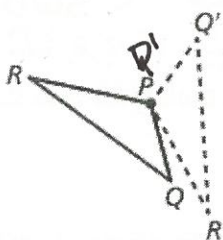
Lesson Objective INBAT perform rotations, perform composition w/ rotations, and identify rotational symmetry.

Rotation: a rotation is a transformation in which a figure is turned about a fixed point called the center of rotation.

→ Rays drawn from the center of rotation to a point and its image form the angle of rotation

A rotation about a point P through an angle of x° maps every point Q in the plane to a point Q' so that one of the following is true:

1. If Q is NOT the center of rotation P , then $QP = Q'P$ and $m\angle QPQ' = x^\circ$
OR
2. If Q is the center of rotation P , then $Q = Q'$



$m\angle APA' = 60^\circ$
 $m\angle BPB' = 60^\circ$

ROTATING COUNTERCLOCKWISE	ROTATING CLOCKWISE
<i>*default direction</i>	

***"about" means "around" in math language

Coordinate Rules for Rotations about the Origin

When a point (x, y) is rotated COUNTERCLOCKWISE about the origin, the following are true:

1. For a rotation of 90° : $(x, y) \rightarrow (-y, x)$
2. For a rotation of 180° : $(x, y) \rightarrow (-x, -y)$
3. For a rotation of 270° : $(x, y) \rightarrow (y, -x)$

**Unless otherwise stated, a rotation will always be counter clockwise

Example:

1. Graph quadrilateral RSTU with vertices $R(3, 1)$, $S(5, 1)$, $T(5, -3)$, and $U(2, -1)$ and its image after a 270° rotation about the origin.

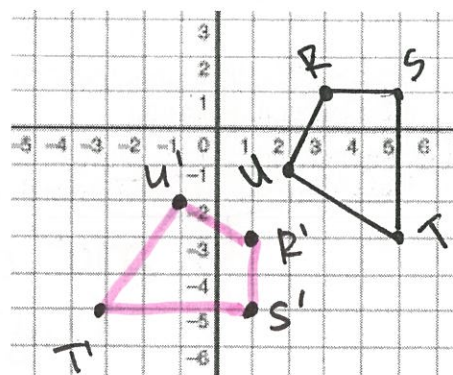
$$(x, y) \rightarrow (y, -x)$$

$$R(3, 1) \rightarrow R'(1, -3)$$

$$S(5, 1) \rightarrow S'(1, -5)$$

$$T(5, -3) \rightarrow T'(-3, -5)$$

$$U(2, -1) \rightarrow U'(-1, -2)$$



****REMEMBER:** a full rotation is 360° .

Rotational Symmetry

A figure in the plane has rotational symmetry when the figure can be mapped onto itself by a rotation of between 0° and 360° about the center of the figure.

→ The point in which the figure rotates is called the center of symmetry

→ Note that the rotation can be either clockwise OR counterclockwise.

→ A figure is said to have point symmetry when the figure has an angle of rotation of 180° .

****Use your trace paper to help you determine whether the figure has rotational symmetry!**

Examples:

2. Determine whether the figure (the star) has rotational symmetry.

yes, a rotation of $< 360^\circ$ maps the $\star$ onto itself.

3. What is the smallest angle that maps the figure (the star) onto itself?

72°

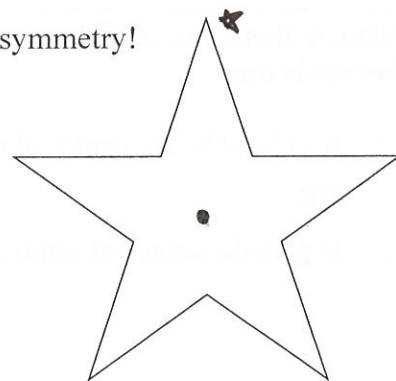
4. Describe what your group did to find the answer to #3.

Divide a full rotation (360°) by the # of times the $\star$ can get mapped onto itself in that 360° rotation.

5. Describe the rotation(s) that map the figure onto itself.

In other words, what angle(s) between 0° and 360° map the figure onto itself?

$72^\circ, 144^\circ, 216^\circ, 288^\circ$



Compositions with Rotations

Example

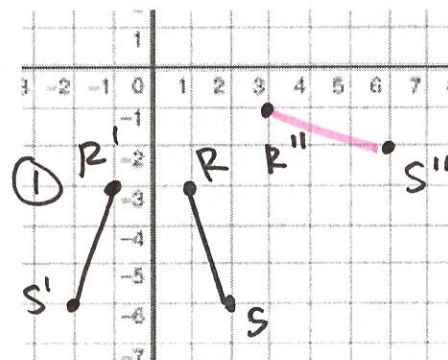
6. Graph $\overline{RS}$ with endpoints $R(1, -3)$ and $S(2, -6)$ and its image after the composition:

① **Reflection:** in the y-axis

② **Rotation:** 90° about the origin

$$R(1, -3) \rightarrow R'(-1, -3) \rightarrow R''(3, -1)$$

$$S(2, -6) \rightarrow S'(-2, -6) \rightarrow S''(6, -2)$$



7. Graph $\overline{AB}$ with endpoints $A(-4, 4)$ and $B(-1, 7)$ and its image after the composition:

① **Translation:** $(x, y) \rightarrow (x - 2, y - 1)$

Rotation: 90° clockwise about the origin

→ 270° counter clockwise
 $(x, y) \rightarrow (y, -x)$

$$A(-4, 4) \rightarrow A'(-6, 3) \rightarrow A''(3, 6)$$

$$B(-1, 7) \rightarrow B'(-3, 6) \rightarrow B''(6, 3)$$

